LEBANESE AMERICAN UNIVERSITY
Department of Computer Science and Mathematics
Calculus IV
Exam III Spring 2011 (May 11, 2011)

Name: Solution **ID:**

SIGN here if you need your grade by Friday____________

Circle your section: 9:30 1:00 MWF @12

Question Number	**Grade**
1. 10%	
2. 10%	
3. 10%	
4. 10%	
5. 10%	
6. 10%	
Total	

No need to complete your integrals.

1. Evaluate the line integral $\int_C \left(-9x^2 + 9e^{9y} + \frac{1}{z+1}\right) ds$,where C is the path $\overrightarrow{r(t)} = \vec{i} + \vec{j} + t\vec{k}$ from $(0,0,0)$ to $(1,1,1)$

$$\begin{cases} x = 1 \\ y = 1 \\ z = t \end{cases}$$

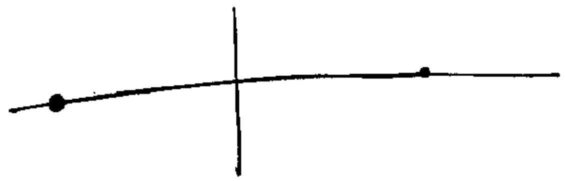
$$\frac{d\vec{r}}{dt} = \vec{k}$$

$$\left|\frac{d\vec{r}}{dt}\right| = 1$$

$$\Rightarrow ds = 1\,dt.$$

$$= \int_0^1 \left(-9 + 9e^9 + \frac{1}{t+1}\right) dt$$

2. Calculate the circulation of the filed $\vec{F} = xy^2\vec{i} + x^2y\vec{j}$ around the curve C which is the segment from $(-7,0)$ to $(7,0)$.



$$\begin{cases} x = t \\ y = 0 \end{cases}$$

$$\vec{r} = t\vec{i}$$

$$\frac{d\vec{r}}{dt} = \vec{i}$$

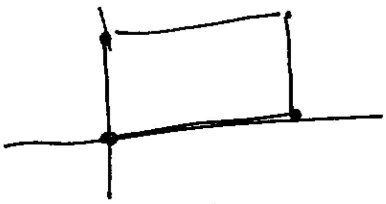
$$\int \vec{F}\cdot\vec{T}\,ds = \int M\,dx + N\,dy$$

$$= \int \left((xy^2)\cdot\frac{dx}{dt} + (x^2y)\frac{dy}{dt}\right)dt$$

$$= 0$$

3. Find the flux of the filed $\vec{F} = xy^2\vec{i} + x^2y\vec{j}$ across the closed plane curve C around the rectangle with vertices $(0,0), (4,0), (4,10)$, and $(0,10)$.

$$\text{Flux} = \int \vec{F} \cdot \vec{n}\, ds = \iint (M_x + N_y)\, dA.$$



$M = xy^2 \Rightarrow M_x = y^2$

$N = x^2y \Rightarrow N_y = x^2.$

$$\boxed{\int_0^4 \int_0^{10} (x^2+y^2)\, dy\, dx.}$$

4. Calculate the flow in the field $\vec{F} = y^5z^3\vec{i} + 5xy^4z^3\vec{j} + 3xy^5z^2\vec{k}$ along the path C which is the line segment from $(5,1,1)$ to $(6,1,-1)$.Note check for the potential of $\vec{F}$ first

$\vec{\nabla} f \overset{?}{=} \vec{F}$ $\quad f_x = M \quad f_y = N \quad f_z = P$

$f = \int M\, dx = \int y^5z^3\, dx = xy^5z^3 + h(y,z)$

$f_y = 5xy^4z^3 + h_y(y,z) = N = 5xy^4z^3$

$\Rightarrow h_y(y,z) = 0 \Rightarrow h = \int h_y\, dy = g(z).$

$\therefore f = xy^5z^3 + g(z)$

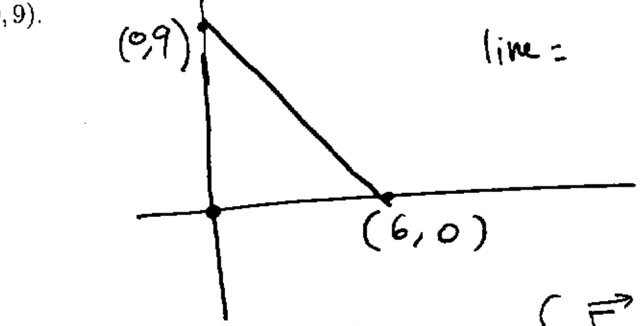
$f_z = P \Rightarrow 3xy^5z^2 + g'(z) = 3xy^5z^2$

$\Rightarrow g(z) = C$

$\Rightarrow f = xy^5z^3 + C$

Answer $= f(6,1,-1) - f(5,1,1) = -6 - 5 = -11$

5. Find the outward flux of $\vec{F} = (x-y)\,\vec{i} + (x+y)\,\vec{j}$ across the triangle with vertices $(0,0)$, $(6,0)$ and $(0,9)$.



line: slope $= \frac{9}{-6} = -3/2$.

$y = -\frac{3}{2}x + 9$

Green thm: Flux $= \int_C \vec{F}\cdot\vec{n}\,ds = \iint (M_x + N_y)\,dA$.

$$= \int_0^6 \int_0^{-3/2x+9} (1+1)\,dy\,dx = \int 2\left(-\tfrac{3}{2}x + 9\right)dx$$

$$= \int (-3x + 18)\,dx = -\frac{3x^2}{2} + 18x \Big|_0^6$$

$$= -3(18) + 18(6) = 54$$

6. Evaluate work done by $\vec{F} = x\,\vec{i} + y\,\vec{j} + z\,\vec{k}$ between $(1,1,1)$ and $(7,5,8)$

$\vec{u} = 6i + 4j + 7k$

$$\int_C \vec{F}\cdot\vec{T}\,ds = \int M\,dx + N\,dy + P\,dz$$

$\vec{r} = (1+6t)\,\vec{i} + (1+4t)\,\vec{j} + (1+7t)\,\vec{k}$ $\quad 0 \le t \le 1$

$\vec{r}\,'(t) = 6i + 4j + 7\vec{k}$ $\quad \frac{dx}{dt} = 6$

$|v| = \sqrt{36+16+49} = \sqrt{101}$ $\quad \frac{dy}{dt} = 4$

$M = x = 1+6t$

$N = y = 1+4t$ $\quad \frac{dz}{dt} = 7$

$P = z = 1+7t.$

$\therefore$ $\boxed{\text{Answer} = \int_0^1 \left[(1+6t)\cdot 6 + (1+4t)\,4 + (1+7t)\,7\right] dt}$